The critical exponents ν, $\nu_{||}$ and $\nu_{\perp}$ of directed SAW on Sierpinski carpets: a computer simulation study

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LETTER TO THE EDITOR

The critical exponents ν, $\nu_\parallel$ and $\nu_\perp$ of directed SAW on Sierpinski carpets: a computer simulation study*

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Abstract. The critical behaviour of fully directed self-avoiding walks (DSAW) on Sierpinski carpets is studied by numerical simulation. We propose a fractal cell generation method to form an infinite fractal lattice with the structure of a Sierpinski carpet. The obtained critical exponent $\nu_\parallel$ is indpendent of the fractal dimension of a Sierpinski carpet d_f but ν and $\nu_\perp$ are dependent on d_f. The results indicate that DSAW on different Sierpinski carpets belong to different universality classes.

The directed self-avoiding walk (DSAW) model was first suggested by Fisher and Sykes [1] in order to derive rigorous bounds for the connectivity constant of isotropic SAW. Critical phenomena of DSAW on Euclidean lattice has been extensively studied since the early days of fractal emergence [2, 3]. The introduction of preferred direction gives rise to two independent correlation lengths $R_\parallel$ and $R_\perp$, parallel and perpendicular to the preferred direction respectively. It is well known that the exponents ν, $\nu_\parallel$ and $\nu_\perp$ for root-mean-square end-to-end displacements, $R^2(N) \sim N^{2\nu}$, $R_\parallel^2(N) \sim N^{2\nu_\parallel}$ and $R_\perp^2(N) \sim N^{2\nu_\perp}$, have values $\nu = \nu_\parallel = 1.0$ and $\nu_\perp = 0.5$ [4]. These are true for any Euclidean dimension $d \geqslant 2$, and also true for directed Levy flight on a square lattice studied by Hu and Yao [5]. We have known that fully DSAW and partially DSAW belong to the same universality class and that two-dimensional DSAW belongs to the same universality class as three-dimensional DSAW, which means that the introduction of the 'direction' plays a dominant role in directed self-avoiding walks [6].

Recently, the problem of random walks on fractal lattices and on percolation clusters has been the subject of much attention [7, 8]. These two problems are related because an exact fractal is a good model for the backbone of the incipient infinite cluster at the percolation threshold. Taguchi [9, 10] studied random walks (RW) and self-avoiding walks (SAW) on Sierpinski carpets. He calculated the fractal dimensions of RW and SAW on Sierpinski carpets, but DSAW have not yet been done. In this letter, using the proposed fractal cell generation method, we study the critical behaviour of DSAW on Sierpinski carpets.

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In order to form an infinite fractal lattice with the structure of a Sierpinski carpet, we propose a method of fractal cell generation. Figure 1 shows an example of the Sierpinski carpets treated in this work. First we construct the first stage of fractal cell (see figure 2(*a*)) which contains eight ($b^2-1^2=8$) lattice sites, then, in the same way, we construct the second stage of fractal cell (see figure 2(*b*)) which contains eight of the first stage cells. The third stage cell (see figure 2(*c*)) can be constructed from the eight second-stage cells in the same way, and so forth. Using this method we can form an infinite fractal lattice with the structure of the Sierpinski carpet shown in figure 1. The meaning of fully directed self-avoiding walks is illustrated in figure 3 where a disallowed step on a Sierpinski lattice is shown.

Figure 4 shows three types of the Sierpinski carpet with the values of d_f as follows, where d_f is the fractal dimension of a carpet:

(*a*) fractal-1, $b=5$, $l=1$, $d_f=1.975$;

(*b*) fractal-2, $b=5$, $l=2$, $d_f=1.892$;

(*c*) fractal-3, $b=4$, $l=2$, $d_f=1.792$.

In order to study the behaviour of DSAW on these fractal lattices we have performed a simulation on our Honeywell DPS8 machine.

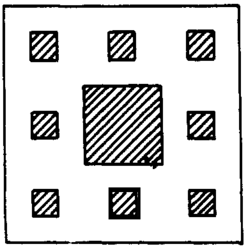

Figure 1. An example of Sierpinski carpets with $b=3$, $l=1$, $d_f=1.893$. The shaded area denotes the hole which was cut off.

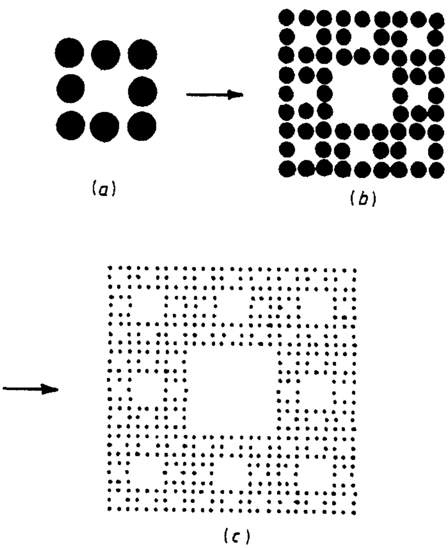

Figure 2. Using the fractal cell generation method the fractal lattice with the structure of Sierpinski carpet as in figure 1 was formed. (*a*) the first-stage cell; (*b*) the second-stage cell; (*c*) the third-stage cell.

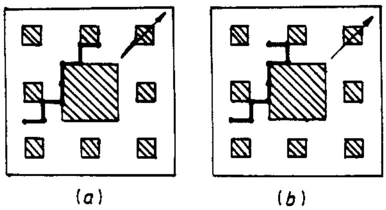

Figure 3. Illustration of the constraint which defines a fully directed self-avoiding walks, including paths of the type illustrated in (*a*) but not those shown in (*b*).

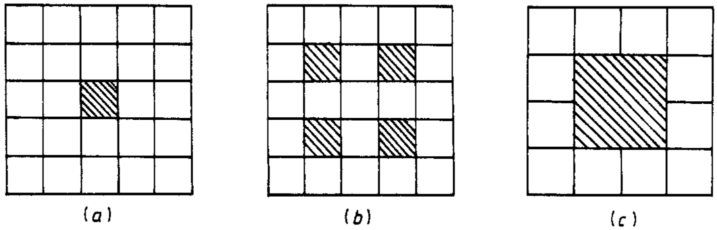

Figure 4. Three types of Sierpinski carpets. (*a*) fractal 1: $b=5$, $l=1$, $d_f=1.975$; (*b*) fractal 2: $b=5$, $l=2$, $d_f=1.892$; (*c*) fractal 3: $b=4$, $l=2$, $d_f=1.792$.

We calculated the correlation lengths $\overline{R^2}$, $\overline{R_\parallel^2}$ and $\overline{R_\perp^2}$ (see figure 5) for N steps in our computer, noting that $\overline{R^2}$, $\overline{R_\parallel^2}$ and $\overline{R_\perp^2}$ imply an averaging over different origins. Expressing $\overline{R^2}$, $\overline{R_\parallel^2}$ and $\overline{R_\perp^2}$ for $N\to\infty$ as:

$$\overline{R^2}\sim N^{2\nu}\overline{R_\parallel^2}\sim N^{2\nu_\parallel}\overline{R_\perp^2}\sim N^{2\nu_\perp}.$$

Simulation results indicate that the relationship between $\overline{R^2}$ (or $\overline{R_\parallel^2}$, $\overline{R_\perp^2}$) and N obeys a good scaling behaviour. Figure 6 gives the plot of $\frac{1}{2}\log_2 \overline{R_\perp^2}$ and $\frac{1}{2}\log_2 \overline{R_\parallel^2}$ against $\log_2 N$ for three Sierpinski carpets. From the slopes of the straight-line fits we obtained the critical exponents for fully directed self-avoiding walks on these Sierpinski carpets:

$d_f=1.975$	$\nu=0.981\pm0.004$	$\nu_\parallel=1.000$	$\nu_\perp=0.59\pm0.01$
$d_f=1.892$	$\nu=0.972\pm0.003$	$\nu_\parallel=1.000$	$\nu_\perp=0.67\pm0.02$
$d_f=1.792$	$\nu=0.971\pm0.007$	$\nu_\parallel=1.000$	$\nu_\perp=0.83\pm0.03$

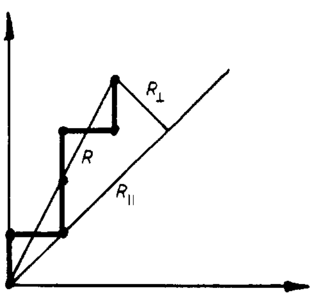

Figure 5. The correlation lengths R, $R_\parallel$ and $R_\perp$ for fully directed self-avoiding walks on Sierpinski carepets.

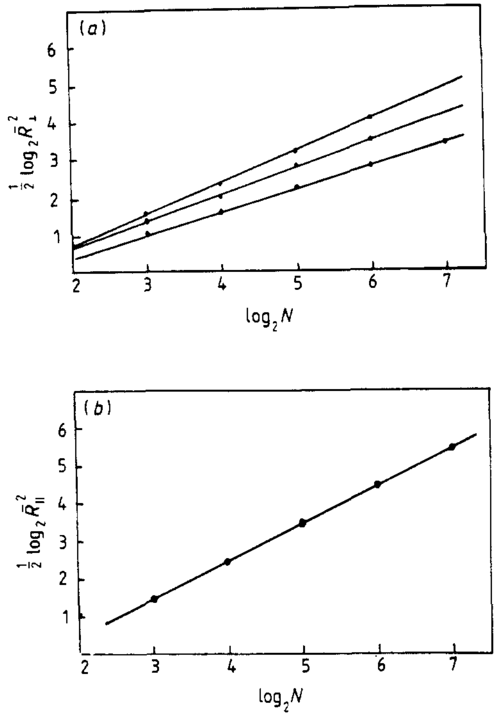

Figure 6. (*a*) Plot of $\frac{1}{2}\log_2 \overline{R_\perp^2}$ against $\log_2 N$ for, from top to bottom, $d_f = 1.792$, 1.892 and 1.975. (*b*) Plot of $\frac{1}{2}\log_2 \overline{R_\parallel^2}$ against $\log_2 N$ for $d_f = 1.792$, 1.892 and 1.795.

From the results we know that ν for directed self-avoiding walks on Sierpinski carpets is different from that on the ordinary Euclidean lattice. It is interesting to note that $\nu_\parallel$ is independent of the fractal dimension d_f, but $\nu_\perp$ is dependent on the fractal dimension d_f (see figure 7). This result suggests that DSAW on Sierpinski carpets do not belong to the same universality class as DSAW on ordinary Euclidean lattices and DSAW on different Sierpinski carpets belong to different unviersality classes.

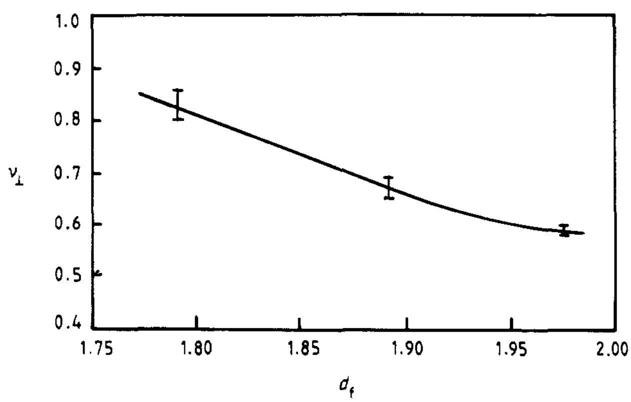

Figure 7. Dependence of $\nu_\perp$ on fractal dimension d_f.

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